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The Influence of Encironment on a Prey-taxis System?

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Abstract

From the dynamic point of view we numerically investigate the influence of predator-prey interactions, the death rates, chemotaxis and diffusivity upon the ability of survival of multiple consumer levels in a closed environment with self-produced nutriment. First we construct a mathematical prey-taxis system to model the real situation. Then we deal with the effects of diffusivity, conversion efficiency and chemotaxis on the predator-prey micro-organism populations and how all the parameters work together to determine the existence of consumer levels by the numerical method.

Keywords: Closed environments; Self-produced nutri-ment; Mathematical models; Existence of consumer levels.

1. Mathematical model

Our model is the following reaction-diffusion system, consisting of a nutrient with the logistical growth and mmicro-organism populations in a closed reactor:

$$\begin{cases} \dfrac{\partial S}{\partial t} = d_0 \Delta S + S(1-S) - u_1 f_1(S), \\ \dfrac{\partial u_i}{\partial t} = d_i \Delta u_i - k_i u_i - div[\Phi_i(u_{i-1}) u_i \nabla u_{i-1}] \\ + u_i f_i(u_{i-1}) - u_{i+1} f_{i+1}(u_i), 1 \le i \le m-1, u_0 = S, \\ \dfrac{\partial u_m}{\partial t} = d_m \Delta u_m - k_m u_m - div[\Phi_m(u_{m-1}) u_m \nabla u_{m-1}] \\ + u_m f_m(u_{m-1}). \end{cases} \tag{1.1}$$

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for $(x,t) \in \Omega \times R^+$, with the boundary conditions

$$\frac{\partial S}{\partial n} = \frac{\partial u_i}{\partial n} = 0, 1 \le i \le m. \tag{1.2}$$

for $(x,t) \in \partial\Omega \times R^+$, and initial data

$$S(x,0) = S_0(x), u_i(x,0) = u_{i0}(x), x \in \bar{\Omega}. \tag{1.3}$$

where Ω is a bounded domain in n-dimensional space (usually, n=3) with the smooth boundary $\partial\Omega$, S denotes the concentration of the nutrient, here the fresh nutrient is produced in the interior of the reactor and is of logistic growth, $u_i(x,t)$ is the density of the i-th micro-organism population, $S_0(x)$ is the concentration of the input nutrient and it is a smooth function and greater than zero on $\partial\Omega, \frac{\partial}{\partial n}$ represents the outward normal derivative on $\partial\Omega$. The nonlinear functions $f_1(S)$ and $f_i(u_{i-1})$ are the functional responses and typically are continuous functions, the derivative term $d_i \Delta u_i$ reflects the random diffusive flux in the model, the term $div(\Phi_i(u_{i-1})u_i \nabla u_{i-1})$ represents the chemotactic flux response of each species to the presence of previous species, the function Φ, the so-called sensitivity rate, is incorporated to indicate the varying sensitivity of cells to the previous level. Many different forms for Φ have been used, as in [1], for example,

$$\Phi(s) = \alpha, \Phi(s) = \frac{\alpha}{(a+s)^2}, \Phi(s) = \frac{\alpha}{a+s},$$

where $\alpha, a > 0$. The boundary condition describes the situation where zero population-flux across the boundary of the closed reactor.

2. Numerical simulation

we will numerically investigate the influence of the con-version rates, the death rates, the diffusion coefficients and sensitivity rate upon the food chain length of a closed system with self-produced nutrient obeying the Logistic growth by direct simulations of the following time-dependent model :

$$\begin{cases} \frac{\partial S}{\partial t} = d_0 \frac{\partial^2 S}{\partial x^2} + S(1-S) - u_1 f_1(S), \\ \frac{\partial u_1}{\partial t} = d_1 \frac{\partial^2 u_1}{\partial x^2} - k_1 u_1 - \frac{\partial}{\partial x}[\Phi_1(S) u_1 \frac{\partial S}{\partial x}] \\ + u_1 f_1(S) - u_2 f_2(u_1), \\ \frac{\partial u_2}{\partial t} = d_2 \frac{\partial^2 u_2}{\partial x^2} - k_2 u_2 - \frac{\partial}{\partial x}[\Phi_2(u_1) u_2 \frac{\partial u_1}{\partial x}] \\ + u_2 f_2(u_1). \end{cases} \tag{2.1}$$

for $x \in (0,1)$, with the boundary condition

$$\begin{cases} \frac{\partial S}{\partial x}|_{x=0} = \frac{\partial S}{\partial x}|_{x=1} = 0, \\ \frac{\partial u_i}{\partial x}|_{x=0} = \frac{\partial u_i}{\partial x}|_{x=1} = 0, i = 1,2. \end{cases} \tag{2.2}$$

and the initial condition

$$S_0(x) = 0.5(\cos(\pi x) + 1), u_{10}(x) = u_{20}(x) = 1. (2.3)$$

We assume that the chemotactic sensitivity follows the receptor law:

$$\Phi_1(S) = \frac{\alpha_1}{(a_1 + S)^2}, \Phi_2(u_1) = \frac{\alpha_2}{(a_2 + u_1)^2}.$$

The response functions are chosen as the following simple functions:

$$f_1(S) = \beta_1 S, f_2(u_1) = \beta_2 u_1,$$

with positive parameters β_1 and β_2. It is easily known that steady states of (2.1)-(2.2) $(\hat{S}_1, \hat{u}_{11}, 0)$ and $(\hat{S}_2, \hat{u}_{12}, \hat{u}_{22})$ are constants .

By trying a wide range of values of the diffusion coefficient and chemotactic parameter for simulation, we observe that the three populations in system (2.1)-(2.3) ultimately stabilize to their own steady states, which shows that diffusivity and chemotaxis do not affect the qualitative behavior of our ecological system. Biologically， population diffusion and the chemotactic flux response of each population to the presence of previous population do not influence the survival ability of population in the self-contained environments with zero population flux across the boundary and homogeneously distribution of population at the area. Therefore, for the next simulations, let the diffusion coefficients $d_0 = d_1 = d_2 = 1$ and chemotactic parameters $\alpha_1 = \alpha_2 = a_1 = a_2 = 1$.

Now we display solution patterns in Fig 1-4 with different values of the conversion rates β_1 and β_2.

Fig.1 depicts the evolution of (2.1)-(2.3) with the parameters $\beta_1 = 0.06$ and $\beta_2 = 0.06$, where it is observed that the washout equilibrium $(S^*, 0, 0) = (1, 0, 0)$ is stable, and the food chain length is 1. Biologically, the conversion efficiency β_1 is so small that the primary consumers can not persist. Naturally the second consumers can not exist even though β_2 is large. The whole space-time region is homogeneously filled with the nutrient.

Fig.2 shows the dynamical behaviors of (2.1)-(2.3) with the parameters $\beta_1 = 0.6$ and $\beta_2 = 0.06$. As seen in the figure, the primary consumers first increase monotonically from 1 to 1.5 mainly due to the stronger conversion efficiency, then decrease oscillatory mainly due to the limited nutrient growth, and finally successfully survive and stabilize to the steady state with average density $\hat{u}_{11} \approx 1.4$; in contrast, the nutrient first decreases monotonically, then oscillates, and finally stabilizes to the uniform concentration $\hat{S}_1 \approx 0.2$ from $0.5(\cos(\pi x) + 1)$; because of the small value of β_2 and low density of the primary consumers, the conversion production from the primary to the secondary consumers is not enough to offset the death of the secondary consumers so that the secondary trophic level becomes extinct, and the food chain length is 2.

Fig.3 displays the development of three populations in system (2.1)-(2.3). We observe that the three populations coexist in our closed bio-reactor, so the food chain length is 3. The second consumers first increase monotonically from 1 to about 1.4 due to their strong eating ability (the large conversion rate β_2), then decrease oscillatory mainly attributed to the limited food and the death, and finally stabilize to a uniform steady state with the density

$\hat{u}_{22} \approx 0.4$. In contrast, the primary consumers first decrease monotonically from 1 to about 0.1 attributed to being largely preyed by the second consumers, then increase oscillatory and end up with a

uniform steady state $\hat{u}_{12} \approx 0.3$. The nutrient first presents oscillation, and then keep the permanent concentration $\hat{S}_2 \approx 0.8$ which is larger than that in Fig.2 attributed to the lower density of the primary consumer than that in Fig.2.

As observed in Fig.4, the different dynamics from Fig.3 is presented by the larger conversion rate $\beta_2 = 4$ than $\beta_2 = 1$ in Fig.3 even though they share the same all other parameters. It is very interesting that the strong consuming ability of the second consumers raises keen competition among the species in our ecosystem; But after a longer-time competition, at last they keep coexistence due to the combined effects of death rates, birth rates, nutrient growth rates and predator-prey interactions.

3. Conclusion

We find that, under certain conditions, diffusivity and chemotaxis do not influence the existence of species in our proposed environment because of the self-contained system with zero population-flux across the boundary. This is a new result different from the observation in [2,3] where the smaller diffusion rate makes species better grow, and also different from the result in [1] where the larger diffusion rate and chemotaxis efficiency are good for species growth. We also find that the species with larger conversion efficiency and smaller death rate enjoys a better survival chance.

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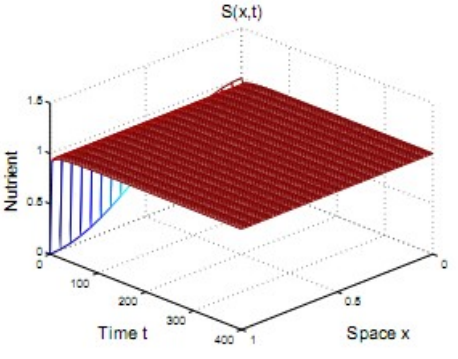
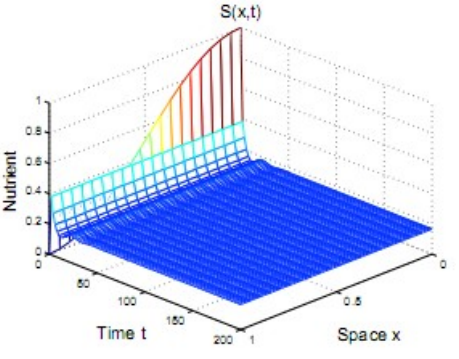




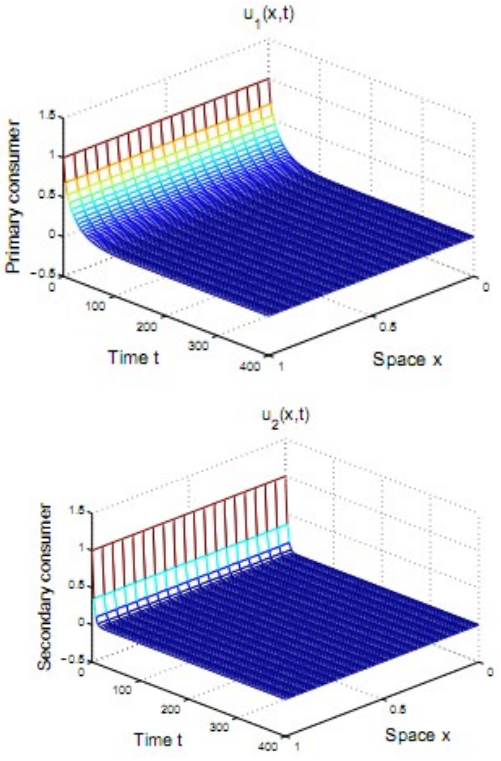


Fig. 1. $\beta_1 = 0.06,\ \beta_2 = 0.06.\ (a).S(x,t),\ (b).u_1(x,t),\ (c).u_2(x,t).$

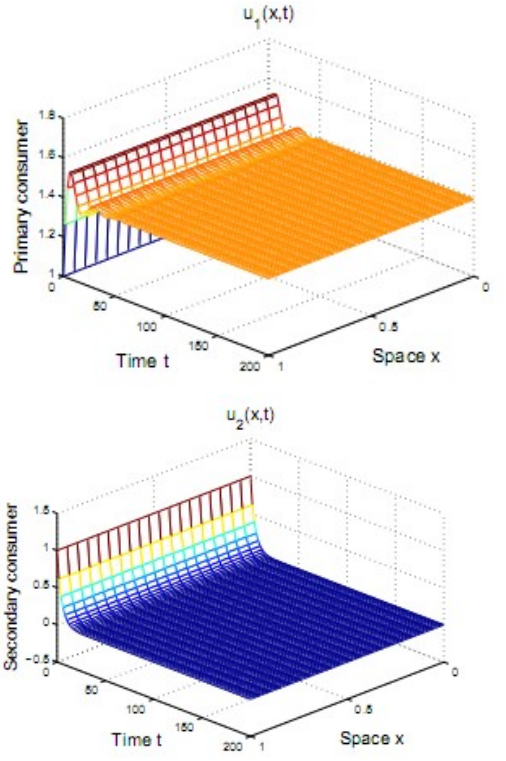
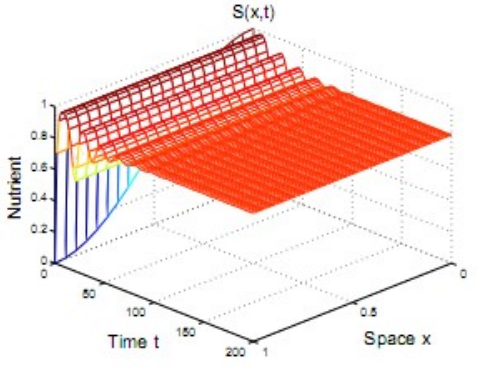
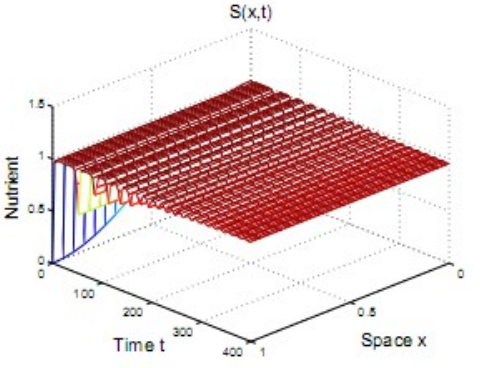


Fig. 2. $\beta_1 = 0.6,\ \beta_2 = 0.06.\ (a).S(x,t),\ (b).u_1(x,t),\ (c).u_2(x,t).$





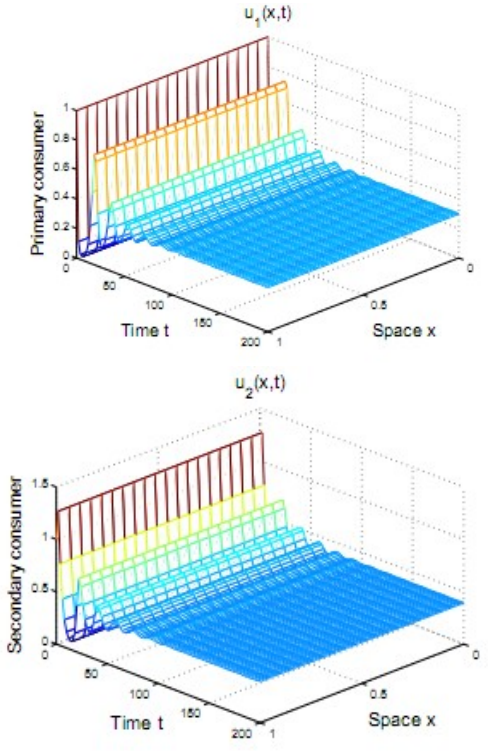


Fig. 3. $\beta_1 = 0.6,\ \beta_2 = 1.\ (a).S(x,t),\ (b).u_1(x,t),\ (c).u_2(x,t).$

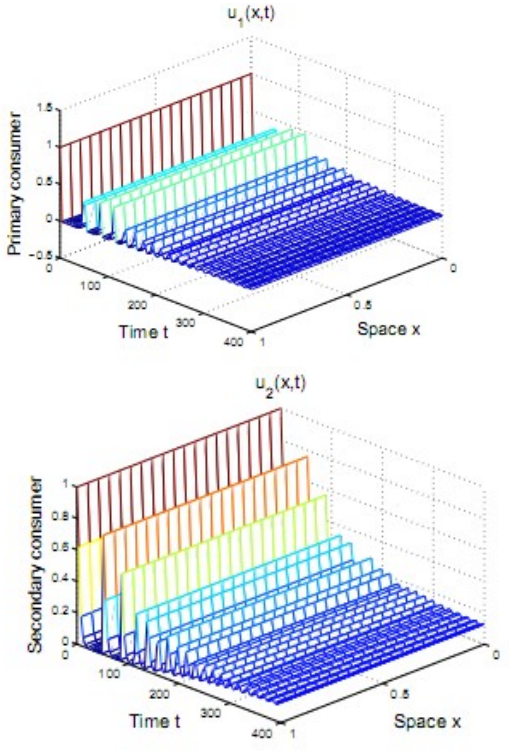


Fig. 4. $\beta_1 = 0.6,\ \beta_2 = 4.\ (a).S(x,t),\ (b).u_1(x,t),\ (c).u_2(x,t).$